STRUDL Modelling Guidelines

Abstract

Attachment A discusses Caltrans STRUDL analysis based on Equal Displacement Principle. Following this background on the correlation between strength reduction factors and displacement ductilities, specific issues related to STRUDL Modelling are discussed, these subjects include:

1. Curved bridges
2. Modelling of superstructure boundaries in long bridges
3. Mode combination
4. Earthquake directions combination
5. Finite size joints
6. Earthquake and live loads combination
7. Soft soil ARS curves
8. Short bridges
9. Torsion/Outriggers
10. New as-built analysis
11. Multi simple-spans bridge modelling

It is important to note that the above subjects can be read separately from each other or from the background discussion on Equal Displacement Principle. Seismic Isolation is being treated separately in Attachment C.

Until recently, Caltrans relied solely on elastic analysis to determine both displacement and moment demands. When moment demand/capacity ratios exceeded a specified limit, the columns were allowed to pin or were retrofit. Currently, Caltrans is placing more reliance on displacement and rotation capacity as outlined in Appendix C to establish column dependability. STRUDL continues to be a reliable method to predict maximum displacements. The following guidelines should be carefully followed when building the analytical bridge model.

STRUDL Modelling

It is generally uneconomical to design bridges to withstand lateral forces corresponding to full elastic response resulting from design-level earthquakes. Caltrans design approach is to accept some seismic damage in a bridge provided it does not lead to the collapse of the structure.

Design requirements will normally dictate that inelastic action occurs in the bridge columns. The reason for this policy is that it is both impractical and undesirable to design for plastic hinges in bridge superstructures, and plastic hinges in piles should be avoided because of difficulties in assessing and repairing damage after an earthquake.

The effect of non-linear behavior on the response of a bridge may be seen with reference to a single degree-of-freedom system. Such a system, responding elastically, will have a load path along O-B (see Figure A1). However, provided sufficient ductility is available and not enough strength to respond elastically, the load path will be along O-A-C (see Figure A1). In fact, a number of comparative linear and nonlinear dynamic analyses have indicated that for long period systems the maximum deflections reached by the two systems are equal {1, 2}. Displacement ductility is defined as:

$$\mu_D = \frac{\Delta\mu(a)}{\Delta y} \qquad \text{(Eq.1)}$$

Since equal displacement principle is considered, the load reduction factor:

$$R = \frac{1}{\mu_D} \qquad \text{(Eq. 2)}$$

Based on that assumption, the structure is designed for an elastic force obtained from STRUDL multiplied by a reduction factor R. (See Figure A1a)

For long period structures, (Eq. 2) is appropriate, implying an 'equal displacement' approach. However, when the structure fundamental period T is less than 0.7 seconds, or when $R < 0.25$ ($Z > 4$), displacements predicted using linear elastic model (STRUDL) may not be appropriate {3, 4}. The following text provides background for this conclusion.

Some nonlinear dynamic analyses indicated that the equal maximum deflection assumption may be unconservative {5}. In particular, reinforced concrete columns show some stiffness degradation from cycle to cycle which result into larger displacement in the nonlinear range. In this latter case, equal energy principle has been shown to be more appropriate. Displacement ductility based on equal energy principle (see Figure A1b) is defined as:

$$\mu_E = \frac{\Delta\mu(b)}{\Delta y} \qquad \text{(Eq. 3)}$$

The reduction factor R in that latter case gives a probable upper value of:

$$R = \frac{1}{\sqrt{2\,\mu_E - 1}} \qquad \text{(Eq. 4)}$$

Figure A2 shows that (Eq. 4) is an upper bound while (Eq. 2) is a lower bound of strength reduction factor R. Part of the problem in equating linear and nonlinear response is the fact that the degraded structure fundamental period increases with time. Although the degraded structure will deflect more with an equal force as compared to the elastic structure, the attracted force becomes less, which would reduce the deflections.

The designer should be reminded that P-Δ effect can be modelled as a form of stiffness degradation. In case of an existing bridge, when a column does not have sufficient strength to resist P-Δ effect, larger displacements are expected. The designer can compute the additional P-Δ displacement by applying to the top of the column an additional lateral force equal to the axial dead load times the elastic demand displacement (STRUDL displacement) divided by the column height. Successive iterations can be performed as described in reference {6} but are not really needed. The computation of this magnified displacement is important to consider when comparing displacement demands to allowable deflections calculated using curvature ductility approach.

Table A1 shows a comparison between μ_D and μ_E values obtained respectively from Eq. 2 and Eq. 4. It can be seen that for a low strength reduction factor equal to 0.17 (i.e., $Z = 6$), the actual demand displacement can be as high as three times the demand displacement calculated by STRUDL. It is important to mention that Eq. 4 gives an upper bound for displacements predicted for a degraded structure. However, Caltrans retrofit philosophy is based on limiting displacement demands over a range where strength loss is not encountered. This is done using maximum allowable flexural ductility ratios or maximum displacement capacity using curvature analysis. Therefore, when eliminating strength loss in the plastic region and maintaining the column's plastic moment carrying capacity, displacement ductility demands based on equal energy principle tend to be overconservative. However, elasto-plastic displacements that are 1.4 to 1.7 times the linear elastic displacements are still possible {7}.

In the absence of nonlinear analysis to predict more accurate displacements, the designer should be reminded that it is possible for the structure to undergo larger displacements than what a STRUDL run reports. Therefore, when comparing STRUDL displacements to allowable deflections obtained from a curvature analysis, a margin of safety should be applied keeping in mind possible displacements magnification discussed earlier.

Although a complete 3-*D* nonlinear analysis is seldom used except as a final check on the adequacy of a completed design, a simplified one degree-of-freedom system can be used to get insight on the correlation between strength reduction factors and displacement ductility ratios. This model is originally referred to as *Q*-model {8} and is based on two types of simplifications:

1. Reduction of a multi-degree-of-freedom model of a structure to a single-degree-of-freedom oscillator,

2. Approximation of the incremental stiffness properties of the entire structure by a single nonlinear spring. In the case of a bridge structure, this spring represents the typical hysteretic behavior of a bridge column. Numerous models are available for that purpose.

The single degree-of-freedom nonlinear analysis is performed from chosen acceleration records and correlation between displacement ductilities and reduction factors is investigated. This type of analysis was completed by CYGNA GROUP/ RICHARD J. STUART, INC. as part of the "Seismic Modelling Parametric Studies" submitted to Caltrans July 1991 {9}. It was found that good correlation exists between Caltrans factors computed on the basis of linear analyses, and ductility demand ratios computed on the basis of nonlinear analyses, for structure periods greater than 0.5 seconds.

For structures' periods less than 0.7 secs, inelastic demand displacements exceed demand displacements predicted using elastic analysis (STRUDL) {10}. The following expression for strength reduction factor R is recommended:

$$\frac{1}{R} = 1 + 0.67\,(\mu - 1)\frac{T}{T_o} \le \mu \quad For \quad \frac{T}{T_o} < 1.5 \qquad \text{(Eq. 5)}$$

where T is the elastic fundamental period of vibration, and T_o is the period corresponding to peak spectral response for the site. The values of T_o are approximately equal to 0.3 sec for Caltrans curves A and B and 0.5 sec for curves C and D. Equation (5) is not used with soft soil spectrum curves (E). Generally, curves E have an ARS plateau ranging from 0.5 to 2.0 seconds. Table A2 shows calculated displacement ductility μ vs. $1/R$. Considering a minimum bridge period of 0.4 sec and the maximum value of T_o equal to 0.5 sec, the minimum ratio of T/T_o is calculated at 0.8. Maximum demand ductility displacements μ correspond to minimum values of T/T_o given a strength reduction factor R. Therefore, a maximum demand displacement magnification ratio of 1.7 (i.e., 10.3/6) is calculated for T/T_o=0.8 and $1/R$ = 6. Generally, for this type of analysis where effective EI are used, bridge periods are lengthened resulting in a higher ratio T/T_o and a smaller demand displacement magnification ratio as shown in Table A2.

In summary, the discussion above illustrates the effects of column stiffness degradation and shorter structures periods resulting in higher inelastic demand displacements than demand displacements calculated using an elastic STRUDL type analysis. Assessment of inelastic displacement demands is essential when comparing these displacements to allowable displacements calculated using curvature analysis approach.

When running STRUDL for estimating displacement demands, effective values of EI for columns should be used. In the absence of a detailed analysis resulting in M-$\emptyset_{flexure}$ diagram (see Figure A3). Effective EI (E: concrete modulus of elasticity; I: flexural moment of inertia) can be used as 0.5 EI gross. Axial stiffness is not usually altered. Furthermore, effective values of GJ (G: concrete shear modulus of elasticity; J: torsional moment of inertia) should be computed. Reference 11 can be used for determining torsional stiffness of diagonally cracked members. In the absence of a detailed analysis, effective GJ value can be considered equal to 0.2GJ gross. Departures from standard procedures should be used with consultation from SASA and presented at strategy meetings.

Following the above description of Caltrans analysis background in using equal displacements principle to correlate strength reduction factor R to displacement ductility demand, some aspects of seismic modelling are discussed to ensure proper use of STRUDL modelling techniques.

1. *Curved Bridges*

In curved bridges, the longitudinal and transverse modes are strongly coupled (i.e., periods of vibration are remarkably close). Curved and Radial Bridges' abutment boundary conditions are not the same as those in straight bridges. Several STRUDL runs are usually performed to bound the bridge complex behavior. As seen in Example 2 of Figure A4, the bridge is not restrained from movement away from the abutments. Therefore, subsequent STRUDL runs turning the abutment stiffness on and off are performed to envelop the structure's response. In case soil anchors or restrainers are present, the two subsequent runs are still performed with abutment tension springs different than abutment compressive springs. It is clear that this latter case is different than the case of straight bridges, where providing a spring stiffness equal to half the abutment compressive stiffness at both ends of the bridge is considered an appropriate approach.

2. *Modelling of Superstructure Boundaries in Long Bridges*

Creating a computer model for the entire length of long bridges is not recommended and produces questionable results, especially because out-of-phase movement is expected in long bridges. STRUDL dynamic modal analysis is based on one phase movement. Additionally, errors may be experienced by not including enough dynamic modes for large computer models (i.e., larger number of nodes). Therefore, it is recommended that the bridge model should not exceed five frames in addition to boundary frames and/or an abutment. Each multi-frame analytical model should be overlapped by at least one useable frame from each model as shown in example 4 of Figure A4.

Boundary frames are frames modelled on either side of the bridge section from which element forces are of interest. They serve as redundant frames in the sense that analytical results are ignored. The use of at least one boundary frame coupled with massless springs at the "dead" end of the model is recommended. The use of boundary frames is an idealization of the structural system. Engineering judgement should be exerted taking into consideration the deformation continuity of various sets of frames. Longitudinal displacements predicted from compression models are used to check the deformation continuity of various sets of frames.

3. *Mode Combination*

The number of modes to be combined in an elastic dynamic analysis is mainly influenced by the number of nodes used to discretize the structure, the frequency content of an earthquake loading, and the structure type or geometry. A straight bridge with single column bents would most probably have a large enough mass participation in one mode (i.e., transverse direction displacement). In contrast, a curved bridge has a much larger modes coupling compared to the straight, thus larger number of modes is necessary to capture more accurate results. In summary, one should include enough modes to capture 90% of the total system mass plus any other modes with relatively large corresponding ARS acceleration. Caltrans approach has been to include a number of modes equal to three times the number of spans. Following this approach, the designer should back check that all bents are excited in the transverse direction. This is done by ensuring that CQC combination includes a minimum number of modes with a maximum transverse normalized displacement of 1 corresponding to each bent. GT STRUDL reports the total mass participation percentage in each direction. This would considerably ease the designer's task of checking the number of modes necessary to be considered in the analysis.

Problems may arise with complex bridges and a more in-depth investigation may be warranted. SASA should be consulted in such matters.

4. *Earthquake Directions Combination*

Analysis and design of bridges should be performed under earthquake loads applied in the direction that results in the structure's "most critical" condition. Finding the most critical direction is an iterative procedure and is time consuming. STRUDL and STRUBAG have rotation capability options that would help the designer in finding the earthquake critical direction.

CYGNA's recommendation on that issue {9} is to use the square root of the sum of squares (SRSS) of any two orthogonal horizontal earthquake forces due to its independence of earthquake orientation. The CYGNA report claims the SRSS combination will be, at most, 12% more conservative than Caltrans linear combination. However, Caltrans' current procedure is still adequate and no final action has been taken on using SRSS combination on a general basis.

Caltrans linear combination of orthogonal seismic forces considers two cases. Case 1 is the sum of forces due to transverse loading Z_G plus 30% of forces due to longitudinal loading X_G ($Z_G + 0.3\, X_G$). Case 2 is the sum of forces due to longitudinal loading X_G plus 30% of forces due to transverse loading Z_G ($X_G + 0.3\, Z_G$). The difference between SRSS and Caltrans linear combination occurs in highly curved bridges where skew angles are as high as 35°. In these bridges, coupling between longitudinal and transverse directions is quite large as compared to straight bridges.

In straight and large radius (i.e.: 3000 and greater) bridges with moderate skew, tension model transverse forces should be combined with compression model longitudinal forces in the conventional ($X_G + 0.3\,Z_G$, $Z_G + 0.3\,X_G$) method. For these types of bridges, it is not rational to expect tension type behavior longitudinally nor for compression type confining effects transversely (see Figure A4).

Vertical earthquake consideration is usually ignored except for outriggers, C-bents, cantilevered sections, and where superstructures are allowed to crack to form a top-of-column pin. For the San Francisco Double-Deck Viaducts a 0.3g vertical acceleration was considered for seismic analysis. A 1.5 factor was used for dead load reactions in Group VII to account for a probable 50% live load and 0.3g vertical excitation. As mentioned earlier, this factor was applied only on Outrigger and C-bent cap beams, and now also applies to top-of-column pins at cracked superstructures.

5. *Finite Size Joints*

Finite size joints should be addressed in structural analysis. It is not uncommon for more than 10% of a bridge column measured from centerlines of joints to be in a rigid zone. If this condition is ignored larger periods may result affecting acceleration levels applied to the structure. Modelling columns to centerline of the bent cap vs. the soffits underpredicts base shears by 17% to 26% {9}. Finite size joints are addressed in STRUBAG by using MEMBER END JOINT SIZE command. However, in this case, forces are reported at centerline of box girder or superstructure and interpolation is needed to get forces at the bottom of the soffit. In order to list forces directly at bottom of soffit, MEMBER ECCENTRICITY command is used.

6. *Earthquake and Live Loads Combination*

The effect of adding live loads to earthquake loads on bridge structures has long since been suspect. However, little investigation has been undertaken to resolve the issue. Some designers believe that it is inappropriate to combine live loads and earthquake loads (e.g., tires and cars will serve as a damping device to the structural system and actually reduce system loads, added axial load due to live load improves column moment capacity, etc.). Loma Prieta drew attention to this issue.

The series of CYGNA studies included a Work Package that considered the effects of the vehicle live load in combination with the seismic forces. This study looked at two cases: Combination 1, dead load ± seismic load (Caltrans seismic load combination) and, Combination 2, dead load + live load ± seismic load. Combination 2 also includes the live load mass excitation. The live load used was two lanes of the HS20-44 lane loading for shear (640 lbs/ft plus 26,000 lb. concentrated load), applied six feet above the bridge deck.

This Work Package indicates that the dead load ± seismic yielded results that are 86% to 100% for column axial loads, and 89% to 92% for shear forces and moments of the results that included the live load.

A more realistic approach taking into account the vehicle type, loading and spacing of vehicles reduces by a factor of three the difference between the two loading combinations including and not including live loads. This latter approach was suggested by Senior Bridge Engineer Earl Seaberg in developing design criteria for Terminal Separation.

In summary, live loads can be ignored in combination with dead load and earthquake loads except for outriggers and C-bents which are designed to force plastic hinging in columns. If plastic hinging is allowed to form in horizontal members, design for shear capacity assuming concrete shear capacity $V_c = 0$ and a vertical shear force due to $L + D + EQ$ loads should be applied.

As mentioned above in section (4), a 1.5 factor was used on San Francisco Viaducts for dead load reactions in Group VII to account for a probable 50% live load and 0.3g vertical acceleration. This factor applies only on Outriggers and C-bent cap beams.

7. *Soft Soil ARS Curves*

The geotechnical branch at TRANSLAB will inform project engineers if soil conditions at a bridge site possess a risk (i.e., liquifaction, lateral spread, amplification of low bedrock acceleration, etc.). However, the project engineer should be alert for warning signs such as liquifaction (saturated sands with blowcounts less than 20) and acceleration amplification (20 feet or greater depths of low blowcounts clays).

In the case of liquefiable material, the designer must investigate the structural response with the liquefiable material in its liquified and unliquified state to ensure structural integrity. Available retrofit options are: 1) to use stone columns, 2) pumps to drain the water, thus reducing pore water pressure, 3) ground injection to stabilize the liquefiable zone, 4) piercing the liquefiable layers with new foundation construction, 5) lower the bridge to ground line, and 6) accept the risk. Such decisions need to be discussed thoroughly with TRANSLAB, management and district representatives.

Caltrans is developing Bay mud spectrum (soft soil). Figure A5 shows an ARS curve for soft soil with 0.5g rock acceleration relative to Caltrans current curves Type B and D. It is recommended that column footing soil springs be used in conjunction with Bay mud spectrum. TRANSLAB has developed soft soil response curves for Terminal Separation, Cypress Viaduct, Southern Viaduct and China Basin. These curves cover a large array of soft soil depths and conditions for designers to use.

Soil spring stiffness for soft soils should be consistent with expected footing displacements for plastic shear forces. The footing/pile system should be modeled in Com 624 and subjected to the expected plastic column shear force. Resulting displacements should match STRUDL ARS footing displacement. Iterate spring stiffness until ARS displacements match Com 624 displacements, within 20%. Even for footings in soft soil, a significant portion of the lateral stiffness is provided by the embedded footing block.

Tests performed on steel piles at the collapsed Cypress Viaduct offer reasonable load-displacement criteria for both soft and dense soils. Values of 30 K/in. lateral capacity, with a maximum displacement of 2 inches, were observed per steel pile for the pile/footing system in soft soils. These values can be used as STRUDL spring modelling criteria. However, the designer should compare the foundation conditions being investigated to the conditions stated in the report {12} and modify values accordingly. Foundation analyses should keep loads and displacement within the safe values determined by the designer. Furthermore, ductile piles for lateral resistance may be required to satisfy the analysis.

8. *Short Bridges*

Short bridges for this discussion are defined as non-skewed or slightly skewed (i.e., < 15°) bridge with no hinges and a length less than 300 ft. In such a structure, the abutment dominates the dynamic response. If the abutments are capable of mobilizing the soil and are well tied into the soil, a damping in the range of 10-15% is justified {13}. The STRUDL library does not presently have damped ARS curves at levels greater than 5%. Therefore, reducing elastic forces and displacements for higher damping effects is possible by applying a damping reduction factor. Considering the short bridge to be a damped single degree-of-freedom system, a reduction factor D can be applied to ARS elastic forces based on the following equation from the Japanese code {14}:

$$D = \frac{1.5}{40c + 1} + 0.5$$

where c = damping ratio

Applying this formula for:

10% damping: forces are multiplied by a factor equal to 0.8,

and 15% damping: forces are multiplied by a factor equal to 0.7.

Generally, the modification factor D for damping should be applied to forces corresponding to the mode that shows the abutment being excited.

In short stiff bridges, emphasis should be placed more heavily on displacements and less on flexural ductility demands when developing a retrofit strategy. Often high flexural demands accompany small column displacements, which means the structure is not subject to collapse.

9. *Torsion/Outriggers*

Torsion is mainly a problem in outriggers connected to columns with top fixed ends. However, torsion can exist in a bent cap beam susceptible to softening due to longitudinal displacements. This softening is initiated when top or especially bottom longitudinal reinforcement in the superstructure is not sufficient to sustain flexural demands due to the localized applied plastic moment of the column. Finding the torsion distribution in the bent cap can be achieved using a grillage analogy that consists of discretizing the deck girders framing into the bent cap beam. Initially, the uncracked torsional stiffness can be adopted and the cap beam investigated under a plastic moment input from the columns under longitudinal response. If the results indicate that the cap beam will crack torsionally over part or all of its length, a second iteration is carried out with the torsional stiffness of the cracking elements reduced to $0.2GJ_g$ where J_g is the uncracked gross torsional moment of inertia and G is the concrete shear modulus. For guidelines on design for torsional forces in outriggers or bent cap beams refer to Attachment B.

Column torsion is not believed to be a problem in single column bents. When taking into account the torsional reduced stiffness, minimal torsional moments are attracted to the columns. Torsional modes are believed to be resisted by coupling of columns' transverse shear forces acting opposite to each other. Torsion is non-existent in multi-column bents.

10. *New As-Built Analysis*

Substantial redistribution of live load may occur in the bridge when as-built conditions of the structure are altered for retrofitting purposes (ex. a pinned column/footing connection is retrofitted with a detail which fixes the connection). It is deemed important that a new as-built analysis be performed for Group Loads I through VI to substantiate bridge load carrying capacity.

11. *Multi Simple-Spans Bridge Modelling*

It is not necessary to use STRUDL for Multi Simple-Span Bridges. A static analysis is fairly adequate since stiffness of cables is much less than columns resulting in very little effects from adjacent frames. Even opposite out-of-phase motion should not cause additional loading. Designers are advised to pay attention to joint details, differential stiffnesses in members, load paths, and restrainer adequacy in their strategy retrofit determination.

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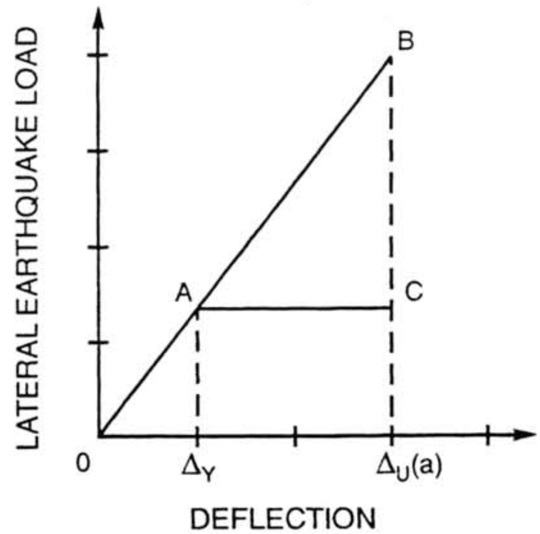
Table A1: Strength Reduction Factors vs. Displacement Ductility

$R = \dfrac{\text{Design Load}}{\text{Elastic Response Load}}$	0.17	0.2	0.25	0.33	0.5	1
μ_D (Eq. 2)	6	5	4	3	2	1
μ_E (Eq. 3)	18.5	13	8.5	5	2.5	1
$\dfrac{\mu_E \text{ (Eq. 3)}}{\mu_D \text{ (Eq. 2)}}$	3.1	2.6	2.1	1.7	1.25	1

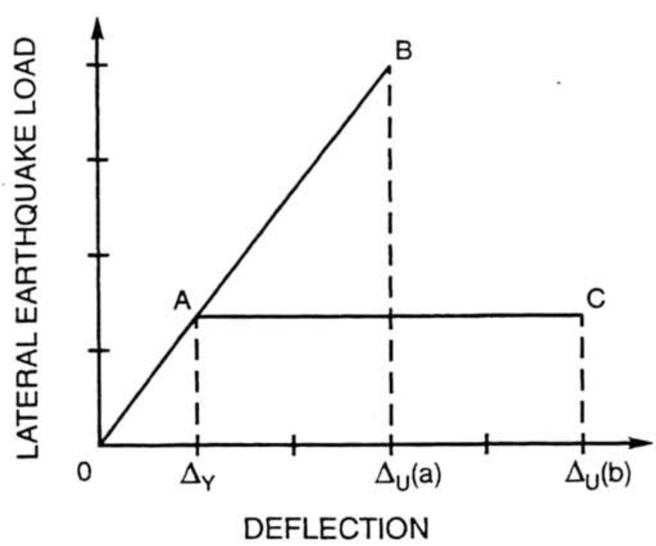
Table A2: Displacement Ductility μ vs. Inverse of Strength Reduction Factors 1/R for Short Periods' Structures.

$1/R$ \ T/T_o	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5
6	10.3	9.3	8.5	7.8	7.2	6.8	6.3	6
5	8.5	7.7	7	6.5	6	5.6	5.3	5
4	6.6	6	5.5	5.1	4.8	4.5	4.2	4
3	4.8	4.3	4	3.7	3.5	3.3	3.1	3
2	2.9	2.7	2.5	2.4	2.3	2.2	2.1	2
1	1	1	1	1	1	1	1	1

$$\frac{1}{R} = 1 + .67(\mu - 1)\frac{T}{T_o} \; for \; \frac{T}{T_o} < 1.5 \quad T_o \left\langle \begin{array}{l} A\ .3 \\ B\ .3 \\ C\ .5 \\ D\ .5 \end{array} \right. \qquad T \left\langle \begin{array}{l} .4\ SEC. \\ .5\ SEC. \\ .6\ SEC. \\ .7\ SEC. \end{array} \right.$$



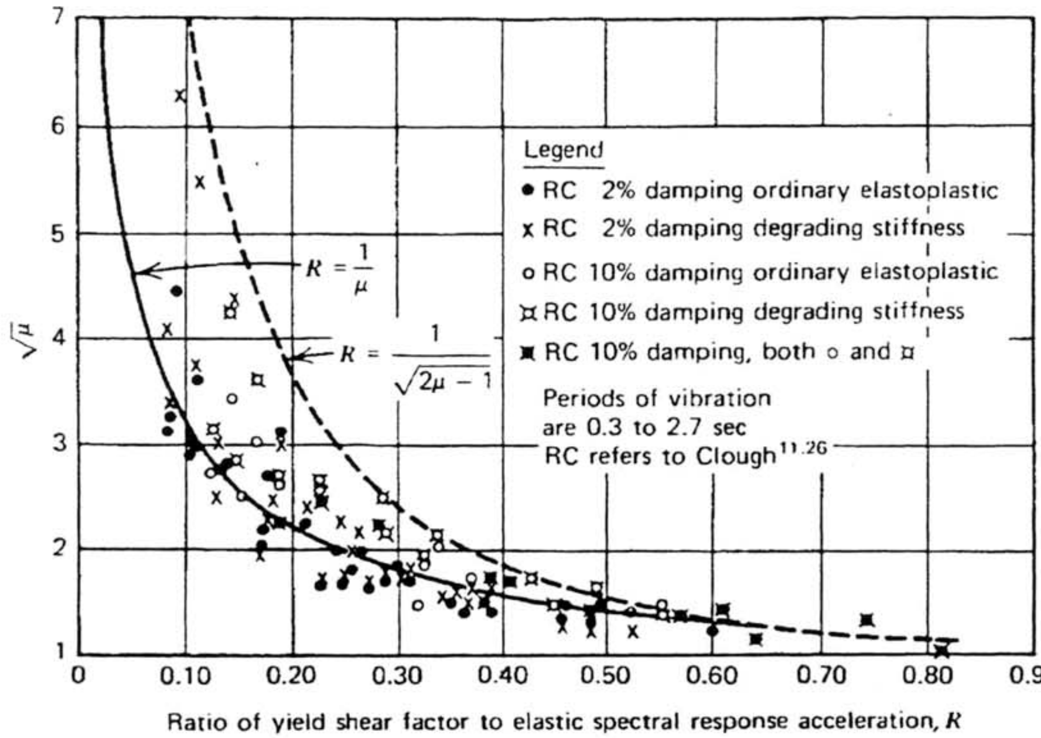
(a) Equal Maximum Deflection



Area 0–B–Δu(a) = Area 0–A–C–Δu(b)

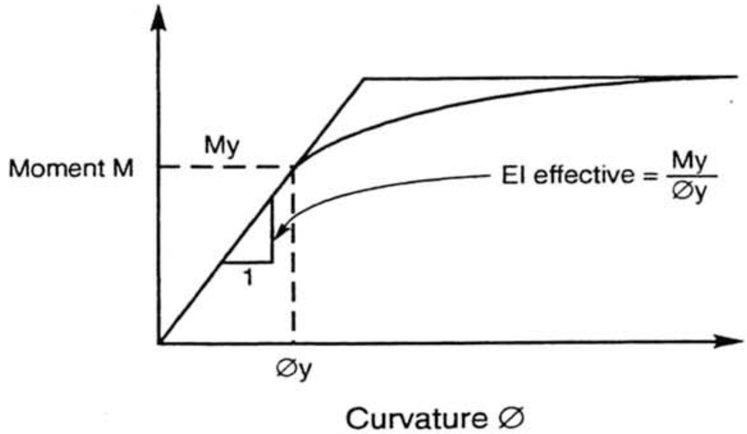
(b) Equal Energy Principle

Figure A1



Displacement Ductility vs. Strength Reduction Factor for Single Degree-of-Freedom Oscillators Responding to the 1940 El Centro N-S Earthquake {5}.

Figure A2



M-∅ Diagram

Note:

1) M - ∅ diagram is needed to calculate allowable deflection using the curvature analysis approach.

2) M - ∅ diagram is dependent on axial force in member. Column dead load can be used in order to estimate *EI* effective.

3) If M - ∅ diagram is not generated, use *EI* effective = 0.5 *EI* gross.

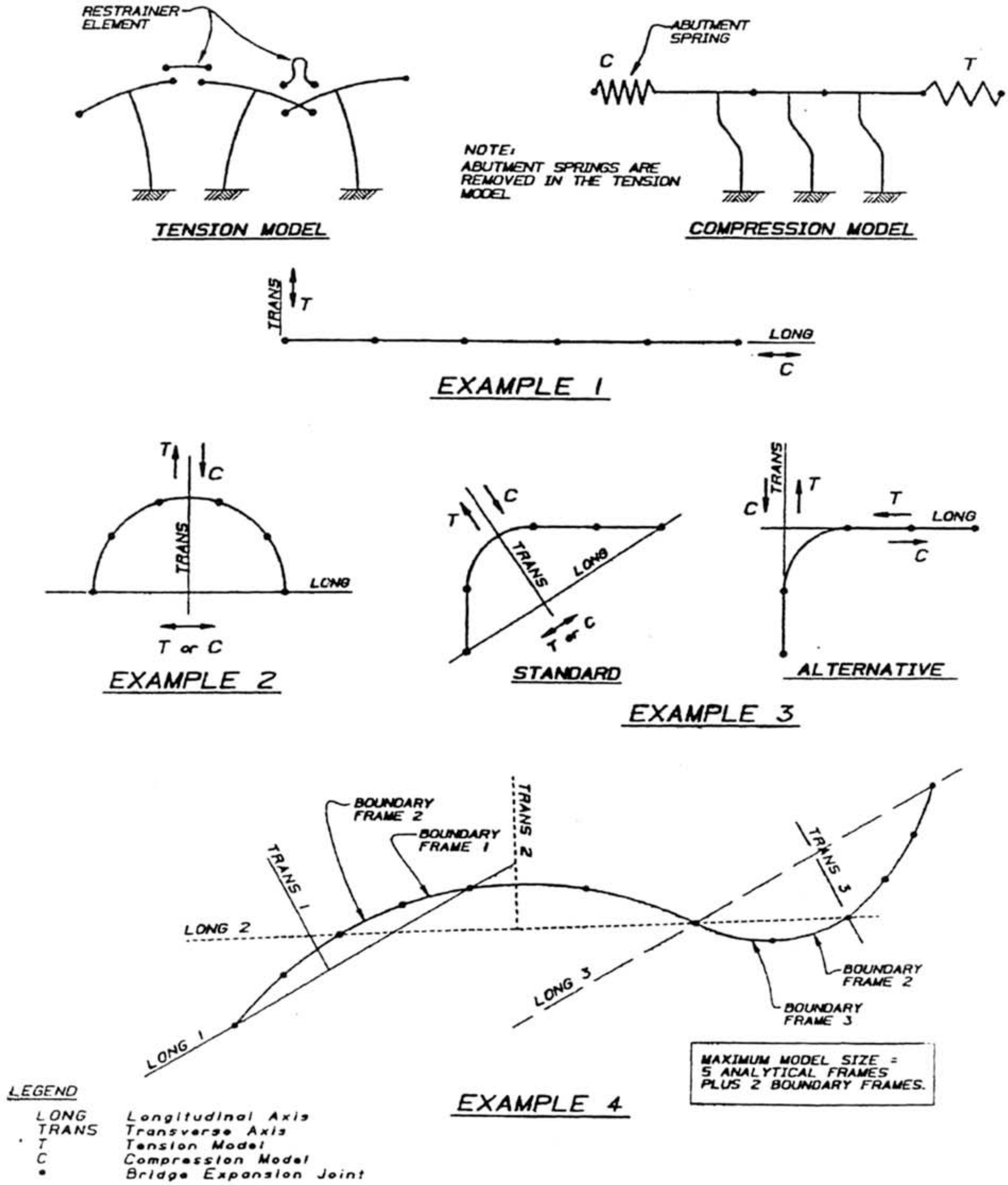
E: concrete modulus of elasticity.

I: flexural moment of inertia.

My: Yield moment capacity.

∅y: Curvature corresponding to first yielding of tensile longitudinal reinforcement.

Moment - Curvature Diagram
Figure A3



Example STRUDL Modelling Techniques

Figure A4

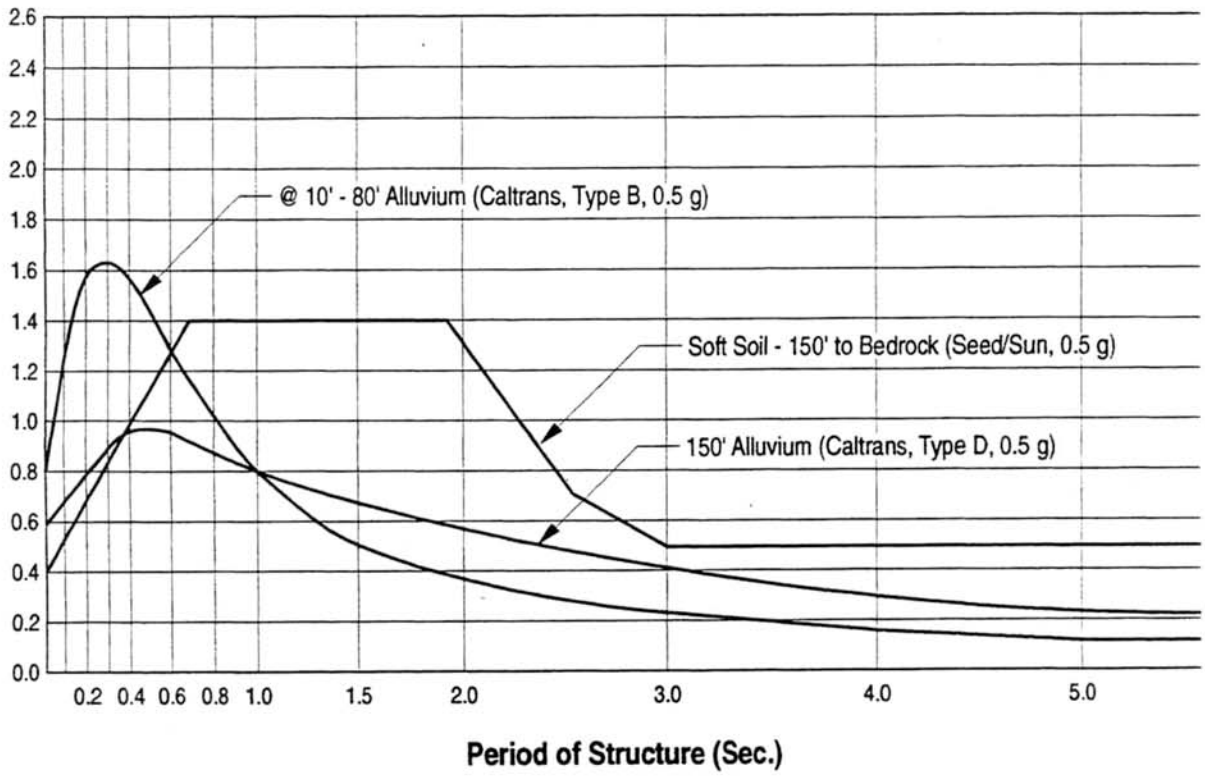

2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
@ 10' - 80' Alluvium (Caltrans, Type B, 0.5 g)
Soft Soil - 150' to Bedrock (Seed/Sun, 0.5 g)
150' Alluvium (Caltrans, Type D, 0.5 g)
0.2 0.4 0.6 0.8 1.0 1.5 2.0 3.0 4.0 5.0
Period of Structure (Sec.)

Figure A5